

Suppression of the Persistent Spin Hall Current by Defect Scattering

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We study the linear response spin Hall conductivity of a two-dimensional electron gas (2DEG) in the presence of the Rashba spin orbit interaction in the diffusive transport regime. When defect scattering is modeled by isotropic short-range potential scatterers the spin Hall conductivity vanishes due to the vertex correction. A non-vanishing spin Hall effect may be recovered for dominantly forward defect scattering.

Spintronics is the rapidly developing field of research aiming to use not only the charge but also the spin degree of freedom of electrons in electronic circuits and devices.¹ In order to be compatible with microelectronic technology, effective spin injection into conventional semiconductors is necessary. Injection of spins via attached ferromagnets has turned out to be quite difficult.^{2,3} This is one motivation to investigate the possibilities to make use of the spin-orbit (SO) interaction, which may spin-polarize a non-magnetic conductor simply by applying a source-drain bias.^{4,5,6,7,8,9} The two-dimensional electron gas (2DEG) is an ideal model system to investigate the physics of these effects. In sufficiently asymmetric confinement potentials the so-called Rashba term dominates the SO interaction.¹⁰ The Datta spin transistor concept¹¹ is based on the tunability of the Rashba interaction by an external gate potential.¹²

Applying an electric field in the x -direction of a Rashba 2DEG spanning the x, y -plane induces a charge current in the x -direction, but also a homogeneous spin accumulation in the y -direction proportional to the field strength.^{6,7} Recently, Sinova *et al.* reported a persistent spin Hall current⁹ for a ballistic Rashba 2DEG. The acceleration of the electrons by the external electric field (along the x -direction) modifies the SO-induced pseudo magnetic field such that spin are tilted out of the 2DEG plane in directions that are opposite for positive and negative lateral momentum (k_y) states. This corresponds to a flow of $s_z = +1/2$ and $-1/2$ spins in opposite directions without a corresponding net charge transport.⁹ The authors suggest that the spin Hall current should be rather robust against disorder scattering, which implies that the effect is measurable in Hall bars of mesoscopic dimensions. Note that the ballistic spin Hall effect is quite different from the spin Hall effect reported earlier for diffuse paramagnetic metals, which is caused essentially by impurity scattering.^{13,14} In the *weak scattering* regime, in which the broadening is smaller than the SO-induced splitting of the energy bands, the life-time broadening of the self-energy has recently been found to have vanish-

ing effect on the ballistic spin Hall current.^{15,16} Burkov and MacDonald¹⁷ recovered the universal ballistic value even in the dirty limit, in which the broadening is larger than the SO-energy splitting. In this Communication we study the effect of disorder on the spin Hall effect in the diffuse regime, in which the scattering rate is larger than either the frequency or the inverse sample traversal time, but for weak scattering. By taking into account the vertex correction we find that the spin Hall effect vanishes identically for short-range impurity scattering.

We model the disorder by randomly distributed isotropic short-range potentials and compute longitudinal and transverse (Hall) conductivities for both charge and spin currents by the Kubo formalism in the Born approximation. The SO interaction is subject to a significant conductivity vertex correction,⁷ which we find here to be decisive for the spin Hall current. The vertex correction appears in such a way that the current operator along the x -direction corresponding to the Rashba Hamiltonian $J_x = e \{(\hbar k/m)\mathbf{1} - \lambda \sigma_y\}$ is modified by substituting $\lambda \rightarrow \tilde{\lambda} = \lambda + \lambda'$. Here σ_i ($i = x, y, z$) are the Pauli spin matrices. The correction term λ' is not necessarily small compared with λ , and found to be $-\lambda$ in the weak scattering regime. Only without the vertex correction, the spin Hall conductivity tends to $e/8\pi$ as predicted by Sinova *et al.*⁹ Physically, the diffuse scattering represented by the vertex correction efficiently scrambles the precession of spins out of the 2DEG plane induced by the applied electric field such that no net spin Hall current remains. On the other hand, the induced spin accumulation in the y -direction is much less sensitive to impurity scattering.⁷ The spin Hall conductivity may persist for long-range, anisotropic defect potentials that correspond to predominantly forward scattering.

The Rashba Hamiltonian in the momentum representation and Pauli spin space reads

$$H_0 = \begin{pmatrix} \frac{\hbar^2}{2m}k^2 & i\lambda\hbar k_- \\ -i\lambda\hbar k_+ & \frac{\hbar^2}{2m}k^2 \end{pmatrix}, \quad (1)$$

where $k_{\pm} = k_x \pm ik_y$ with $\mathbf{k} = (k_x, k_y)$ the electron mo-

mentum in the 2DEG plane, and λ parametrizes the tunable spin-orbit coupling. The eigenfunctions and eigenvalues of the Hamiltonian corresponding to periodic boundary conditions are given as

$$\phi_{\mathbf{k}s} = \frac{1}{\sqrt{2L^2}} e^{i\mathbf{k}\cdot\mathbf{r}} \begin{pmatrix} is\frac{k_-}{k} \\ 1 \end{pmatrix}, \quad (2)$$

$$E_{ks} = \frac{\hbar^2 k^2}{2m} + s\lambda\hbar k, \quad (3)$$

respectively, with $s = \pm$, $k = \sqrt{k_x^2 + k_y^2}$ and L^2 the area of the 2DEG. The corresponding (free) Green function is denoted as $g_{ks}(z)$ with an energy z on the complex energy plane.

The disorder is modeled as randomly distributed, identical defects with point scattering potentials that are neither spin-dependent nor flip the spin:

$$V(\mathbf{r}) = V\mathbf{1} \sum_i \delta(\mathbf{r} - \mathbf{R}_i). \quad (4)$$

which gives rise to an isotropic (s -wave) scattering of electrons. The configurational averaged Green function reads

$$\tilde{G}(k\pm) = \frac{1}{z - E_{k\pm} - \Sigma_{k\pm}(z)}. \quad (5)$$

In the Born approximation the self-energy $\Sigma(z)$ is a state-independent constant:

$$\Sigma(z) = \frac{nV^2}{2L^2} \sum_{ks} g_{ks}(z), \quad (6)$$

where n is the impurity concentration and L denotes the linear dimensions of the sample. The self-energy is related to the scattering life time τ via $|\text{Im}\Sigma| = \hbar/2\tau$ at the Fermi energy ϵ_F .

The charge current operator in spin space reads¹⁸ $J_x = e\partial H_0/\partial p_x = e(bk_x\mathbf{1} - \lambda\sigma_y)$ and $J_y = e\partial H_0/\partial p_y = e(bk_y\mathbf{1} + \lambda\sigma_x)$ with $b = \hbar/m$. The spin currents are represented by the Hermitian operators⁹

$$J_\alpha^{\sigma_i} = \frac{\hbar}{4} \{v_\alpha, \sigma_i\} = \frac{\hbar}{4} \left\{ \frac{\partial H_0}{\partial p_\alpha}, \sigma_i \right\}, \quad (7)$$

where $\alpha = x, y$, and z . Thus $J_x^{\sigma_x} = (\hbar/2)bk_x\sigma_x$, $J_x^{\sigma_y} = (\hbar/2)(-\lambda\mathbf{1} + bk_x\sigma_y)$ and $J_x^{\sigma_z} = (\hbar/2)bk_x\sigma_z$, whereas $J_y^{\sigma_x} = (\hbar/2)bk_y\sigma_x$, $J_y^{\sigma_y} = (\hbar/2)bk_y\sigma_y$, and $J_y^{\sigma_z} = (\hbar/2)bk_y\sigma_z$.

The Kubo formula for the longitudinal electrical conductivity can be written

$$\sigma_{xx} = \frac{\hbar}{2\pi L^2} \text{Tr} \langle J_x G J_x G \rangle_{AV}. \quad (8)$$

where the trace is taken over wave vectors and band index. We evaluate $\langle J_x G J_x G \rangle_{AV} = J_x \langle G J_x G \rangle_{AV} \equiv J_x K_x$

in the ladder approximation that obeys the Ward relation with the self-energy in the Born approximation. This leads to the Bethe-Salpeter equation

$$K_x \approx \tilde{G} J_x \tilde{G} + \tilde{G} \langle V K_x V \rangle_{AV} \tilde{G}. \quad (9)$$

$K_x = \tilde{G} \tilde{J}_x \tilde{G}$ has the same structure as $\tilde{G} J_x \tilde{G}$, and⁷

$$\tilde{J}_x = e \left(b k_x \mathbf{1} + \frac{\tilde{\lambda}}{k} (k_x \sigma_z - k_y \sigma_y) \right). \quad (10)$$

with $\tilde{\lambda} = \lambda + \lambda'$. The vertex correction λ' is the solution of

$$\lambda' = \frac{nV^2}{4L^2} \sum_{k_1} [b k_1 G_{k_1}^- + (\lambda + \lambda') (G_{k_1}^+ + G_{k_1}^{+-} + G_{k_1}^{-+})], \quad (11)$$

with $G_k^s = G_k^{++} + sG_k^{--}$ and $G_k^{ss'} = \tilde{G}_{ks} \tilde{G}_{ks'}$.

The generalized spin conductivity tensor in Pauli spin space reads

$$\sigma_{\alpha x}^{\sigma_i} = \frac{\hbar}{2\pi L^2} \text{Tr} J_\alpha^{\sigma_i} \langle G J_x G \rangle_{AV} \sim \frac{\hbar}{2\pi L^2} \text{Tr} J_\alpha^{\sigma_i} K_x, \quad (12)$$

where the vertex function is the same as before. Symmetry tells us that

$$\text{Tr} J_x^{\sigma_x} K_x = \frac{e\hbar b^2}{8} \text{Tr} k^2 G^+ \sigma_x + \frac{e\hbar b \tilde{\lambda}}{8} \text{Tr} k G^- \sigma_x, \quad (13)$$

$$\begin{aligned} \text{Tr} J_x^{\sigma_y} K_x &= \frac{e\hbar b^2}{8} \text{Tr} k^2 G^+ \sigma_y + \frac{e\hbar b (\tilde{\lambda} + \lambda)}{8} \text{Tr} k G^- \sigma_y \\ &\quad - \frac{e\hbar b \lambda \tilde{\lambda}}{8} \text{Tr} [G^{+-} - G^{-+}] \sigma_y, \end{aligned} \quad (14)$$

$$\text{Tr} J_x^{\sigma_z} K_x = \frac{e\hbar b^2}{8} \text{Tr} k^2 G^+ \sigma_z + \frac{e\hbar b \tilde{\lambda}}{8} \text{Tr} k G^- \sigma_z, \quad (15)$$

$$\text{Tr} J_y^{\sigma_x} K_x = \frac{e\hbar b \tilde{\lambda}}{8} \text{Tr} k (G^{+-} - G^{-+}) \sigma_y, \quad (16)$$

$$\text{Tr} J_y^{\sigma_y} K_x = -\frac{e\hbar b \tilde{\lambda}}{8} \text{Tr} k (G^{+-} - G^{-+}) \sigma_x, \quad (17)$$

$$\text{Tr} J_y^{\sigma_z} K_x = i \frac{e\hbar b \tilde{\lambda}}{8} \text{Tr} k (G^{+-} - G^{-+}) \mathbf{1}. \quad (18)$$

Because the Green functions depend only on k , the angular averages of k_x^2 and k_y^2 are $k^2/2$, and odd terms with respect k_x and k_y in the trace of the equations above vanish by symmetry. Without SO interaction, all matrix elements of $J_\alpha^{\sigma_i} K_x$ vanish except for $J_x^{\sigma_x} K_x = J_x^{\sigma_y} K_x = J_x^{\sigma_z} K_x$. But also these terms become zero after taking the trace. This means that no spin current flows along the external electric field.⁷ Only the spin Hall conductivity $\sigma_{yx}^{\sigma_z}$ proportional to $\text{Tr} J_y^{\sigma_z} K_x$ is nonzero, indicating that a spin Hall current along the y -direction and polarized in the z -direction may exist when an external electric field is applied along x , as predicted for the ballistic limit.⁹

The magnitude of the spin Hall effect can be calculated easily by adopting the following approximation for the

product of Green functions at the Fermi energy ϵ_F ,

$$\tilde{G}(ks)\tilde{G}(ks) \approx \frac{2\pi\tau}{\hbar}\delta(\epsilon_F - E_{ks}), \quad (19)$$

which holds when the energy dependence of the self energy is weak and the broadening is small compared to the SO energy splitting at the Fermi energy, $|\text{Im}\Sigma| \ll 2\hbar\lambda k$. Then

$$\sigma_{yx}^{\sigma_z} = -\sigma_{xy}^{\sigma_z} = \frac{e\tilde{\lambda}}{8\pi\lambda}, \quad (20)$$

for $\epsilon_F > 0$. This agrees with the ballistic result $\sigma_{yx}^{\sigma_z} = e/8\pi$ by Sinova *et al.*⁹ except for a factor $\tilde{\lambda}/\lambda = 1 + \lambda'/\lambda$ due to the vertex correction, but is identical to it when the vertex correction λ' is neglected.

By substituting Eq. (19) into Eq. (11), the vertex correction λ' is evaluated as $\lambda' = -\lambda$, *i.e.* the spin Hall conductivity vanishes. Eq. (19) is equivalent to the weak scattering or strong SO interaction limit. As far as the spin Hall current is concerned, the effect of the impurity vertex correction is thus found to be much more important than that of the impurity self-energy in the Green function.

The ballistic result can be recovered by considering the frequency dependent conductivity

$$\sigma_{\mu\nu}^{\xi} = \lim_{\omega \rightarrow 0} \frac{Q_{\mu\nu}^{\xi}(\omega) - Q_{\mu\nu}^{\xi}(0)}{-i\omega}, \quad (21)$$

in terms of the correlation function

$$\begin{aligned} Q_{\mu\nu}^{\xi}(i\nu_{\ell}) &= \frac{1}{L^2\beta} \sum_m \text{Tr} [J_{\mu}^{\xi}G(i\omega_m + i\nu_{\ell})J_{\nu}G(i\omega_m)] \\ &= \frac{1}{L^2} \text{Tr} J_{\mu}^{\xi} K_{\nu}(i\nu_{\ell}), \end{aligned} \quad (22)$$

with

$$K_{\nu}(i\nu_{\ell}) = \frac{1}{\beta} \sum_m \langle G(i\omega_m + i\nu_{\ell})J_{\nu}G(i\omega_m) \rangle_{\text{AV}}. \quad (23)$$

The vertex correction is calculated as before resulting in

$$K_x(i\nu_{\ell}) = \frac{1}{\beta} \sum_m \tilde{G}(i\omega_m + i\nu_{\ell}) \tilde{J}_x \tilde{G}(i\omega_m) \quad (25)$$

where \tilde{J}_x includes $\tilde{\lambda} = \lambda + \lambda'(\omega)$ with

$$\lambda'(\omega) = -\frac{\hbar}{\tau} \frac{\lambda}{-i\hbar\omega + \frac{\hbar}{\tau}}, \quad (26)$$

by letting $i\nu_{\ell} \rightarrow \hbar\omega + i0$. Here we invoked again the weak scattering assumption.¹⁹ This result generalizes Eq. (20). When the $\tau \rightarrow \infty$ limit is taken first, $\lambda'(\omega) \rightarrow 0$, thus recovering the ballistic limit.⁹ When we take the $\omega \rightarrow 0$ limit first, $\lambda'(\omega) = -\lambda$, and the spin Hall conductivity vanishes as before.

We made the rather crucial approximation that the scattering potential is short-ranged, thus isotropic in momentum space. As mentioned above, Sinova *et al.*⁹ explain the ballistic spin Hall current in terms of the precession of spins out of the 2DEG plane when accelerated by the electric field. In the presence of isotropic impurity scattering, electrons with momentum \mathbf{k} are scattered into all other momenta \mathbf{k}' at the Fermi energy with equal rate, and the spin Hall current disappears with the average spin tilting. This picture is not appropriate anymore when the impurity potentials are long-ranged, scattering predominantly in the forward direction. In that case the short-range model misrepresents the “skew scattering” corresponding to a non-zero Hall angle.²⁰

For long-range anisotropic scatterers the longitudinal conductivity is governed not by the energy life time τ but the transport (momentum) life time τ_t because the momentum integration in the vertex function over $V^2 k_x$ (x is the current direction) does not vanish.¹⁹ Physically this means that the forward (small angle) scattering does not contribute to the resistivity. Without SO interaction, the vertex correction due to anisotropic scattering reads

$$b' = \frac{\langle nV^2 \rangle}{2L^2} \frac{1}{\kappa} \sum_{k_1} (b + b') G_{k_1}^+, \quad (27)$$

where $b = \hbar/m$, $1/\kappa = \tau/\tau_1$, and $\langle nV^2 \rangle$ is an average of the scattering potential over Fermi surface. The transport life time is given by $1/\tau_t = 1/\tau - 1/\tau_1$.

When the SO interaction is incorporated into this vertex correction, the expression of the longitudinal charge conductivity and spin accumulation obtained before⁷ are modified as

$$\sigma_{xx} = 2 \left[\frac{e^2 \tau_t}{m} n_0 + e^2 D \tau_t \lambda^2 \right], \quad (28)$$

and

$$\langle s_y \rangle = 2\tau_t e E D \lambda, \quad (29)$$

respectively. Here we have used following relations: $1/\tau = 2\pi n V^2 D / \hbar = n V^2 m / \hbar^3$, with $D = m/2\pi\hbar^2$, where D is the density of states of 2DEG. Note that the relation $\sigma_{xx}^{\uparrow\downarrow} = \sigma_{xx}^{\downarrow\uparrow}$ holds for arbitrary value of b' and λ' .

The spin Hall effect may survive when small angle scattering dominates because only states close to each other in momentum space are scrambled. The anisotropy may affect the effective current operator in Eq. (11): the first term in parenthesis on the right-hand side becomes λ in the isotropic scattering case and is likely to dominate for not too large long-range potentials. The vertex correction λ' is then given by

$$\lambda' = \frac{\langle nV^2 \rangle}{4L^2} \frac{1}{\kappa'} \sum_{k_1} b G_{k_1}^-, \quad (30)$$

in which k_{1x}^2 and $k_{1y}^2 = k_1^2 - k_{1x}^2$ are replaced with weighted averages $k_1^2/2\kappa'$ and $k_1^2 - k_{1x}^2/2\kappa'$ over the angle. With $1/\kappa' = \tau/\tau'$, we get $\tilde{\lambda} = (\tau/\tau_H)\lambda$ and

$1/\tau_H \equiv 1/\tau - 1/\tau'$. In the isotropic case, $\tau_H \rightarrow \infty$, and $\tilde{\lambda} \rightarrow 0$, but in general the spin-Hall current is finite. This argument does not take into account the full effects of the anisotropy but demonstrates how the vertex correction for anisotropic scattering affects the spin-Hall conductivity.

Burkov and MacDonald¹⁷ computed the spin Hall conductivity for the Rashba-2DEG model system with short-range impurity scattering, but focusing on the dirty limit in which the lifetime broadening exceeds the SO energy splitting, opposite to the clean limit discussed here. Surprisingly, they recover the universal ballistic value found by Sinova *et al.*⁹ This implies that with increasing (short-range) impurity scattering the Hall conductivity first vanishes, but in a reentrant fashion increases again when entering the dirty regime, in which Eq. (19) does not hold anymore.

Murakami *et al.*^{8,21} developed a theory for the spin Hall currents in hole-doped semiconductors described by the Luttinger Hamiltonian. Separating the spin Hall current into a topologically conserved (intraband) and non-conserved (interband) contribution,²¹ these authors contend that the former, which does not exist in the Rashba-2DEG, is robust against impurity scattering.²²

The breakdown of the spin Hall current by impurity scattering in the Rashba-2DEG discussed here would then correspond to the vanishing of the non-conserved part of the spin-Hall current. Microscopic calculations for the Luttinger Hamiltonian analogous to the present ones are necessary to unambiguously prove that the topological spin Hall current indeed survives under impurity scattering.

In conclusion, we have examined the effect of impurities on the spin Hall conductivity of a Rashba-split 2DEG and found that the vertex correction (diffuse electron scattering) to the conductivity is essential, causing the spin-Hall effect to vanish.

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- ¹ S.A. Wolf, D. D. Awschalom, R.A. Buhrman, J.M. Daughton, S. von Molnar, M.L. Roukes, A.Y. Chtchelkina, and D.M. Treger, *Science* **294**, 1488 (2001).
- ² R. Fiederling, M. Keim, G. Reuscher, W. Ossau, G. Schmidt, A. Waag, and L. W. Molenkamp, *Nature* **402**, 787 (1999).
- ³ Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura, H. Ohno, and D. D. Awschalom, *Nature* **402**, 790 (1999).
- ⁴ F. T. Vas'ko and N. A. Prima, *Sov. Phys. Solid State* **21**, 994 (1979).
- ⁵ L.S. Levitov, Yu.V. Nazarov and G.M. Eliashberg, *Zh. Eksp. Teor. Fiz.* **88**, 229 (1985).
- ⁶ V.M. Edelstein, *Sol. Stat. Commun.* **73**, 233 (1990).
- ⁷ J. Inoue, G.E.W. Bauer, and L. W. Molenkamp, *Phys. Rev. B* **67**, 033104 (2003).
- ⁸ S. Murakami, N. Nagaosa, and S-C. Zhang, *Science* **301**, 1348 (2003).
- ⁹ J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, cond-mat/0307663.
- ¹⁰ E.I. Rashba, *Fiz. Tverd. Tela* **2**, 1224 (1960) [Sov. Phys. Solid State **2**, 1109 (1960)]; Yu. A. Bychkov and E.I. Rashba, *Pis'ma Zh. Eksp. Teor. Fiz.* **39**, 66 (1984) [JETP Lett. **39**, 78 (1984)].
- ¹¹ S. Datta and B. Das, *Appl. Phys. Lett.* **56**, 665 (1990).
- ¹² J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, *Phys. Rev. Lett.* **78**, 1335 (1997).
- ¹³ J.E. Hirsch, *Phys. Rev. Lett.* **83**, 1834 (1999).
- ¹⁴ S. Zhang, *Phys. Rev. Lett.* **85**, 393 (2000).
- ¹⁵ J. Schliemann and D. Loss, cond-mat/0310108.
- ¹⁶ N. A. Sinitsyn, E. M. Hankiewicz, W. Teizer, and J. Sinova, cond-mat/0310315
- ¹⁷ A. A. Burkov and A. H. MacDonald, cond-mat/0311328
- ¹⁸ L. W. Molenkamp, G. Schmidt, and G. E. W. Bauer, *Phys. Rev. B* **64**, 121202(R) (2001).
- ¹⁹ G. D. Mahan, *Many-Particle Physics* (Kluwer Academic, N.Y. 2000).
- ²⁰ C.-N. Chazalviel, *Phys. Rev. B* **11**, 3918 (1975).
- ²¹ S. Murakami, N. Nagaosa, and S-C. Zhang, cond-mat/0310005.
- ²² S-C. Zhang, private communication.